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Power Perturbation Method for Power Flow Analysis

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(Abstract) This paper proposesan improved load flow method based on perturbation theory which is simple, efficient and reliable. The performance of the proposed method has been tested on well-conditioned IEEE 5-bus, 14-bus, 30 bus and also tested on 11-bus ill conditioned test systems. The proposed method shows its efficacy and robustness with minimum number of iterations along with high accuracy on conventional load flow methods like G-S (Gauss-Seidel) method.

Keywords: Acceleration Factor; Gauss-Seidel (G-S) Method; Power Perturbation (P-P) Method; Specified Real & Reactive Power.

1. INTRODUCTION

Load flow calculations are imperative for operational planning, control, system stability and reliability. The load flow study [1]-[4] is an important tool involving numerical analysis applied to a power system. Study of load-flow studies is very important as present electrical power system which is continuously expanding in size and growing in complexity. This study ultimately gives important conclusions while to determine the steady state operation of an Electric power system.

The principal information obtained from the power flow study [5]-[6] is real & reactive power flowing in each line, voltage magnitude, phase angle at each bus. The development of digital solution can be found in [7]-[9]. There are various methods are available and some modifications on the existing methods are shown in [10]-[11]. In this paper a simple, efficient and reliable method based on Power Perturbation has been described which involve lower storage requirement along with advantages like convergence and accuracy of conventional techniques. As a result it can also be used in real time applications.

2. LOAD FLOW PROBLEM

It is convenient to represent power system networks using the single line diagram, which can be thought of as the circuit diagram of the per-phase equivalent. In general, representation of a single line diagram is used for load flow studies.

Figure 1 shows the schematic diagram of the basic structure of a typical bus system. It consists of n number of lines with lumped parameters like line voltages. V_1, V_2, ….V_n and line admittances like Y_{i1}, Y_{i2},…..Y_{in} respectively and Power node Equations and subsequent Mathematical model for the said problem can be enumerated as follows:

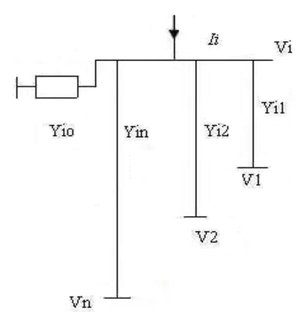

Figure 1: Basic schematic diagram of a typical bus system.

i) Current injections at any node *n* can be written as,

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + ... + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + ... + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - .... - y_{in}V_n \end{aligned} \quad (1)$$

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j-1}^{n} y_{ij}V_j \quad (2)$$

ii) The real and reactive power at bus i is

$$P_i + jQ_i = V_i I_i^* \quad (3)$$

$$I_i = [(P_i - jQ_i)/V_i^*] \quad (4)$$

Power flow equations formulated in polar form. For the system in Fig.1, Eqn.2 can be written in terms of bus admittance matrix as:

$$I_i = \sum_{j=1}^{n} Y_{ij}V_j \quad (5)$$

Expressing in polar form:

$$I_i = \sum_{j=1}^{n} |V_{ij}||V_j| \angle \theta_{ij} + \delta_j \text{........}(6)$$

Substituting for I_i from Eqn.6 in Eqn. 4

$$P_i - jQ_i = |V_i| \angle -\delta_i \sum_{j=1}^{n} |V_{ij}||V_j| \angle \theta_{ij} + \delta_j \text{.....}(7)$$

Separating the real and imaginary parts,

$$P_i = \sum_{j=1}^{n} |V_i||V_j||V_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)\text{............}(8)$$

$$Q_i = \sum_{j=1}^{n} |V_i||V_j||V_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)\text{.............}(9)$$

(

Expanding Eqns. 8 & 9 by Taylor's series results to

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(k)}}{\partial \delta_2^{(k)}} \cdots \frac{\partial P_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial P_2^{(k)}}{\partial |V_2|} \cdots \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \frac{\partial P_2^{(k)}}{\partial \delta_2^{(k)}} \cdots \frac{\partial P_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial P_2^{(k)}}{\partial |V_2|} \cdots \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \hline \frac{\partial Q_2^{(k)}}{\partial \delta_2^{(k)}} \cdots \frac{\partial Q_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} \cdots \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \frac{\partial Q_2^{(k)}}{\partial \delta_2^{(k)}} \cdots \frac{\partial Q_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} \cdots \frac{\partial Q_2^{(k)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \Delta \delta_n^{(k)} \\ \hline \Delta |V_2|^{(k)} \\ \Delta |V_n|^{(k)} \end{bmatrix} \tag{10}$$

3. GAUSS SEIDEL METHOD

The Gauss-Seidel method [12]-[14] is an iterative process which starts by assigning estimated values to the unknown bus voltages. Using the estimated bus voltages and the specified real and imaginary power values, a new value for each bus voltage is calculated at the end of each iteration. The process is repeated until the difference between each bus voltage and its corresponding value in two successive iterations is less than a predefined tolerance value.

For a total of N buses, the calculated voltage at any bus K is expressed as a function of the real and reactive power delivered to a bus from generators or supplied to the load connected to the bus, the estimated or previously calculated voltages at the other buses, and the self- and mutual admittances of the nodes.

Equation (5) is an algebraic non linear equation which must be solved by iterative techniques and V_i can be obtained by solving iteratively.

$$V_i^{k+1} = \frac{\frac{P_i^{sp} - jQ_i^{sp}}{V_i^{*(k)}} + \sum y_{ij} V_j^{(k)}}{\sum y_{ij}}, \; j \neq i \tag{11}$$

Where y_{ij} is the actual admittance in p.u. the specified real & reactive power (P_{isp}&Q_{isp}) are in p.u. by assuming current entering bus I was positive. Thus for Generator buses these injected real and reactive powers have positive values. For Load buses the real and reactive powers flow away from the bus, P_{isp} and Q_{isp} have negative values. By solving Equation (5) the P_i and Q_i are

$$P_i^{k+1} = R\left(V_i^{*(k)} [V_i^{*(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} - V_j^{(k)}]\right\} j \neq i \tag{12}$$

$$Q_i^{k+1} = -I_m \left(V_i^{*(k)} [V_i^{*(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} - V_j^{(k)}]\right\} j \neq i \tag{13}$$

The power flow equation is usually expressed in terms of the elements of the bus admittance matrix, Y_{bus}, shown by upper case letters, are $Y_{ij} = -y_{ij}$, and the diagonal elements are $Y_{ii} = \sum y_{ij}$. Therefore Equation (6) can be written as

$$V_i^{k+1} = \frac{\frac{P_i^{sp} - jQ_i^{sp}}{V_i^{*(k)}} + \sum Y_{ij} V_j^{(k)}}{\sum Y_{ij}}, \; j \neq i \tag{14}$$

$$P_i^{k+1} = R\left(V_i^{*(k)} [V_i^{*(k)} \sum_{j=0}^{n} Y_{ij} - \sum_{j=1}^{n} Y_{ij} - V_j^{(k)}]\right\} j \neq i \tag{15}$$

$$Q_i^{k+1} = -I_m \left(V_i^{*(k)} [V_i^{*(k)} \sum_{j=0}^{n} Y_{ij} - \sum_{j=1}^{n} Y_{ij} - V_j^{(k)}]\right\} j \neq i \tag{16}$$

Iterative Stages for the above method is as follows:

- Slack bus: both components of the voltage are specified. 2(n-1) equations to be solved iteratively.
- Flat voltage start: initial voltage of 1.0+j0 for unknown voltages.
- PQ buses: P_{isp} and Q_{isp} are known. with flat voltage start, Equation (9) is solved for real and imaginary components of Voltage.
- PV buses: P_{isp} and [Vi] are known. Equation (11) is solved for Q_i^{k+1} which is then substituted in Equation (9) to solve for V_i^{k+1}

However, since [V_i] is specified, only the imaginary part of V_i^{k+1} is retained, and its real part is selected in order to satisfy

$$(e_i^{(k+1)})^2 + (f_i^{(k+1)})^2 = |V_i|^2 \tag{17}$$

Or, $$e_i^{(k+1)} = (|V_i|^2 - (f_i^{(k+1)})^2)^{1/2} \tag{18}$$

Acceleration factor: It is the rate of convergence is increased by applying an acceleration factor to the approximate solution obtained from each iteration.

$$V_i^{(k+1)} = V_i^{(k)} + \alpha\,(V_{ical}^{(k)} - V_i^{(k)}) \tag{19}$$

Iteration is continued until

$$|e_i^{(k+1)} - e_i^{(k)}| \le \varepsilon, \;\; |f_i^{(k+1)} - f_i^{(k)}| \le \varepsilon \tag{20}$$

Once a solution is converged, the net real and reactive powers at the slack bus are computed from Equations (10) & (11).In the original algorithm of Gauss-Seidel method the voltage at each bus at any iteration is calculated by using the voltages of previous buses calculated at the same iteration and the voltages of the next buses calculated at the previous iteration i.e. where the difference of corrected voltage precision index and previous voltage is within the acceptable limit. The entire process is carried out again and again until the amount of correction in

voltage at every bus is less than some predetermined precision index.

It has been professed that the G-S method for solving large power flow problems takes excessive number of iterations and more computational time. However the proposed Power Perturbation method shows that the total execution time is considerably reduced.

4. POWER PERTURBATION METHOD

Very often, a mathematical problem cannot be solved exactly or, if the exact solution is available, it exhibits such an intricate dependency in the parameters that it is hard to use as such. Perturbation theory gives a systematic answer to this question. The proposed methodology can be outlined by the flowchart as shown in Figure 2

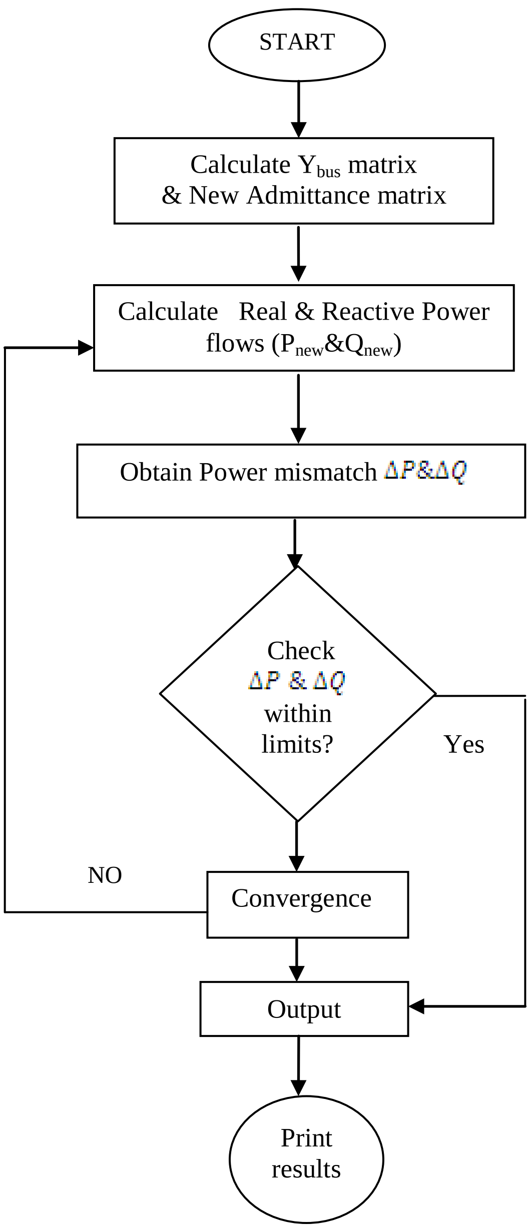

Figure 2. Procedure of Power perturbation method

Perturbation theory [15]-[18] comprises mathematical methods that are used to find an approximate solution to a problem which cannot be solved exactly, by starting from the exact solution of a related problem. Also this theory leads to an expression for the desired solution in terms of a power series in some "small" parameter that quantifies the deviation from the exactly solvable problem. The leading term in this power series is the solution of the exactly solvable problem, while further terms describe the deviation in the solution, due to the deviation from the initial problem. Formally, we have for the approximation to the full solution A, a series in the small parameter (here called ε), like the following: $A=\varepsilon^0 A_0+\varepsilon^1 A_1+\varepsilon^2 A_2+\ldots\ldots$

A_0 would be the known solution to the exactly solvable initial problem and $A_1,A_2,...$ represent the higher order terms which may be found iteratively by some systematic procedure. For small ε these higher order terms in the series become successively smaller. An approximate "perturbation solution" is obtained by truncating the series, usually by keeping only the first two terms, the initial solution and the "first order" perturbation correction: $A\approx A_0+\varepsilon A_1$

5. PERFORMANCE ANALYSIS

The performance of the P-P Method was tested on IEEE 5-bus, 14-busand 30-bus as well as on ill conditioned bus system [19] with a convergence accuracy of 10^{-3} on a MVA base of 100 or equivalently 10^{-1} MVA for both power residuals ΔP and ΔQ. It requires the same storage size as the Gauss-Seidel method, but less computation time than the conventional Gauss-Seidel program. It is attractive for accurate or approximate results and contingency calculations for IEEE test systems. The proposed solution algorithm can be implemented directly and efficiently into microcomputers with limited storage capacities.To examine the effectiveness, test systems including IEEE 5-bus, 14-bus, 30-bus and 11-bus ill conditioned bus systems were studied by the proposed method, and the convergence characteristics of these bus systems are compared with the Gauss-Seidel method.

5.1 Number of Iteration

The Gauss-Seidel method of solving large power flow problems it has been seen that an excessive number of iterations are required for different IEEE test systems compared to the proposed Power-Perturbation method before the voltage corrections are within an acceptable precision index. The test results are given in Table 1 and the corresponding graph is plotted in Figure 3. The power flow solution based on power perturbation method shows its robustness and assured convergence for all types of test systems with minimum number of iterations and also it is very important that P-P Method converged for an ill conditioned system (11-bus) where conventional G-S Method diverged.

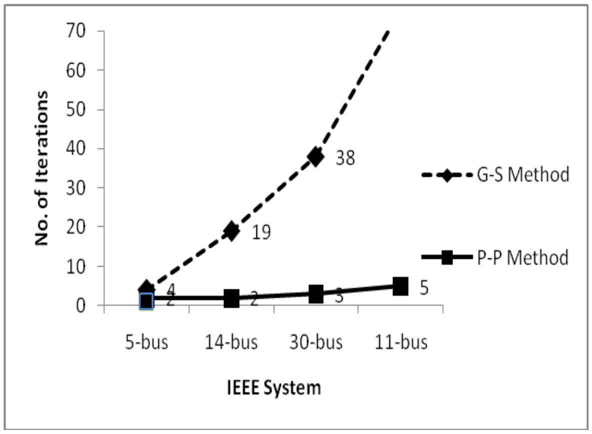

Figure 3.Comparison of G-S & P-P Method (Iterations for different IEEE bus systems)

5.2 Computation Time

The algorithm has been developed and run on Intel core 2 duo processor, 1.86 GHz, 2 GB RAM, 250 GB of Hard-disc laptop using MATLAB 7. The results of time-comparison are given in Table 2. In conventional Gauss-Seidel based power flow algorithm, Jacobian matrix is required.

As memory requirement of matrixes is more, CPU time per generation for conventional method is also **more. The power perturbation tec**hnique shows better 'time per iteration' compared to the conventional Gauss-Seidel method.

The convergence time for G-S and proposed P-P method are shown in Table 1, where it is clearly depicted that G-S Method fails to converge for ill conditioned system where as P-P method gives the solution.

The same is shown in the Figure 4 in a graphical form which reveals that time required for Power Perturbation method is smaller than the conventional G-S method.

Table1.Comparison of Computation Time in Second

IEEE Bus	G-S	P-P
5-bus	0.04	0.01
14-bus	0.367	0.08
30-bus	1.13	0.09
Ill condition Bus	d*	1.017

d*= Divergent

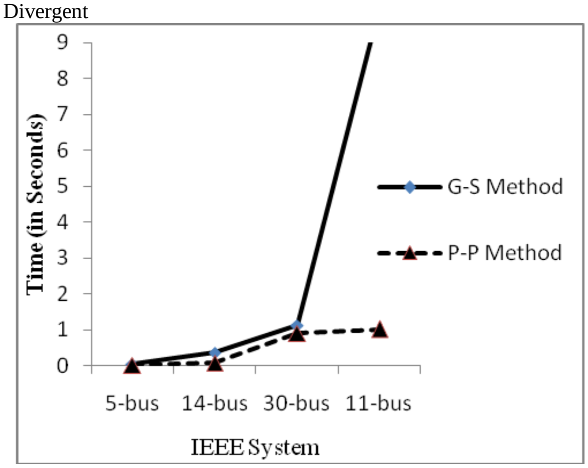

Figure 4 Comparison of G-S & P-P Method (Convergence time in seconds for different IEEE bus systems)

5.3 Accuracy

Both G-S & P-P methods were tested by varying accuracy from 10^{-1} to $10^{-3}$the and was found that the Gauss- Seidel method will have maximum number of iterations compared to the proposed Power-Perturbation method as shown in Table 2.

Table 2. Comparison of Accuracy and Iterations

	GS			P-P		
Accuracy	10^{-1}	10^{-2}	10^{-3}	10^{-1}	10^{-2}	10^{-3}
IEEE 5-bus	4	5	9	1	2	3
IEEE 14-bus	19	43	22	1	2	3
IEEE 30-bus	14	25	34	2	5	8

6. CONCLUSION

Power Perturbation method gives faster, smarter and more accurate results pertaining to the conventional Gauss-Seidel technique of load flow calculation.Test results of the proposed method is performed on the well-conditioned IEEE 5-bus, 14-bus and 30-bus system as well as on ill conditioned 11-bus system.

The numerical results show that the P-P method is indeed feasible, effective and is superior to the G-S method. Further research work can be done on Power-Perturbation method for more complex bus system as well as ill-conditioned systems to verify its ability of convergence, robustness and the pace at which it is providing the solution.

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